

Thm contd.

III If L is a CFL, then L^R is also a CFL.

Let L be generated by a CFG G . We specify a CFG G' for L^R .

G' : If $A \Rightarrow \alpha$ is in G , then $A \Rightarrow \alpha^R$ is in G' .

Example: $G: S \Rightarrow aSb \mid ab$

Then $G': S \Rightarrow bSa \mid ba$

Proof of correctness is not needed; but the following is a proof.

We prove that for every $A \in V_N$ & $\alpha \in \Sigma^*$, $A \xrightarrow{G} \alpha$ iff $A \xrightarrow{G'} \alpha^R$ by induction on the length of derivation. For simplicity, we assume that G is in the normal form.

Base: For one step derivation, clearly

$$A \xrightarrow{G} \alpha \text{ iff } A \xrightarrow{G'} \alpha^R$$

Let the claim hold for $\leq k$ step derivations.

For any $k+1$ step derivation, ~~\Rightarrow~~ $A \xrightarrow{G} BC \xrightarrow{G} \alpha\beta$ in which B & C derive α & β , respectively, in no more than k steps.

$$\begin{aligned} \text{By ind. hyp } B \xrightarrow{G} \alpha &\text{ iff } B \xrightarrow{G'} \alpha^R \text{ & } C \xrightarrow{G} \beta \text{ iff } \\ C \xrightarrow{G} \beta^R. \text{ Hence } & \end{aligned}$$

$$A \xrightarrow{G} BC \xrightarrow{G} \alpha\beta \text{ iff } A \xrightarrow{G'} CB \xrightarrow{G'} \alpha^R\beta^R.$$

Hence the ind. step holds.

Hence, for every $x \in \Sigma^*$,

$$S \xrightarrow{G} x \text{ iff } S \xrightarrow{G'} x^R.$$