600.271 Key to HW 7
Sp 2012

I (a) PCP $(x_1, y_1), \ldots, (x_n, y_n)$ each $|x_i|$ & $|y_i|$ even

will ~~reduce~~ PCP $\leq_m$ this problem

Typical inst. $(x_1, y_1) \ldots (x_n, y_n)$ $(u_1, v_1), \ldots, (u_m, v_m)$ s.t. $|u_i|, |v_i|$ even
P P'

* Given P, transform it to P' s.t. P has a solution iff P' has a solution.

Replace each (x_i, y_i) by (u_i, v_i) by doubling each symbol in x_i & y_i. For example $(001, 10)$ is replaced by $(000011, 1100)$.

It is easily seen that (*) is satisfied. In addition, the transformation is a computable function

(d) dlba long infinite?

We transform ~~the~~ BT halting problem $\leq_m$ this problem

Typical inst $[M]$ $[M']$

* Given $[M]$, transform it to $[M']$ s.t. TM M halts on BT iff dlba M' accepts an infinite set of strings.

M': On any input $x = a_1 a_2 \ldots a_n$, M' erases x and starts simulation of M on BT. During the simulation it cannot go outside the n squares. If M' detects that M has halted, then M' accepts x. In all other cases (M is in a loop, or M needs more than n squares), M will not accept x.

halts. We argue that (*) is satisfied.

If M halts on BT, then let it halt making use of T squares of space. Then observe that M' accepts every string of length $\geq T$. (M' also accepts shorter strings & are of the form $a^i b^i c^i$). Hence $L(M')$ is a CFL.

If M doesn't halt on BT, ~~M' accepts~~ then $L(M') = \{a^n b^n c^n \mid n \geq 1\}$ which is not a CFL.

Hence (*) is satisfied.

Also, the ~~transformation~~ $[M]$ to $[M']$ is computable.

Also transformation of $[M']$ to an equivalent CSG is computable.

(9) TM M halts on some input & doesn't halt on some input.

We show BT HP $\leq$ this problem

Typical inst $[M]$ $[M']$

* Given TM M, transform it to TM M' s.t. M halts on BT iff M' halts on input 0 and doesn't halt on input 1.

M': On input 1, M' goes into an infinite loop (i.e. doesn't halt.

On input 0, M' erases 0 & simulates M on BT.

On any other input, it doesn't matter. Let us say that M' halts.

Now we show that (*) is satisfied.

If M halts on BT, then let it halt in T steps.

Then M must not have used more than T squares.

So any input of length $\geq T$ will be accepted by M'. That is $L(M') = \cancel{\{0,1\}^T\{0,1\}^*}$, an infinite $\{x \mid x \in \{0,1\}^*, |x| \geq T\}$, an infinite set.

If M doesn't halt on BT, M' cannot accept any string; i.e. $L(M') = \phi$, a finite set.

Hence (*) is satisfied.

The transformation from [M] to [M'] is computable.

(f) whether a given CSG generates a CFL.

We show BT halting problem $\leq_m$ this problem.

Typical insts [M] CSG G' or nlba M'

Given a TM M, transform it to nlba M' (it will be a dlba) s.t. M halts on BT iff $L(M')$ is a CFL.

M': given an input from $\cancel{\{0,1,2\}^*}$ $\{a,b,c\}^*$, it ~~will~~ check whether the input is of the form $a^n b^n c^n$. If so it ~~will~~ ~~reject~~ accepts the string. If not, it ~~will~~ erases the 3n squares & starts simulation of M on BT. As ~~before~~ in the previous example, M' accepts the input if it detects that M

The condition (*) is satisfied.

The transformation is computable.

II It is true that the condition
M halts on input x iff $c = a + b$ is satisfied.

However the transformation is not computable.

If M doesn't halt on x, then no output is specified ~~to~~ since the computation has not terminated.

(other researchers follow a slightly different definition of computability. When you ~~can~~ produce a part of the output, you cannot change that output. That is, one assumes that the output has to be written on a one-way output tape.)