600.271 Automata & Computation Theory
Final Examination
December 12, 2009
In-class, Closed Book
Time: 2 hrs 30 mins.

All the subproblems carry equal weight.

I. Design an nfa for the language:
$\{xcycz \mid x, y, z \in \{a, b\}^*,\ (\#_a x (\text{mod } 2) = |z| (\text{mod } 2)) \text{ or } (\#_b x (\text{mod } 2) = |y| (\text{mod } 2))\}$.

II. Design a CFG (i.e. type 2 grammar) for the language:
$\{xcycz \mid x, y, z \in \{a, b\}^*,\ (y = x^R \text{ and } \#_a z \text{ is even}) \text{ or } (y = z^R \text{ and } |x| \text{ is odd})\}$.

III. Prove that the following problems are decidable.

1. Given an npda M and a string x, is $x \in L(M)$?

2. Given a dlba M and a positive integer T, does there exist an input string x which is accepted by M within T steps?

IV. Prove that the following set S is recursively enumerable by providing an appropriate enumeration algorithm. For any given Post Correspondence Problem $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$, let $S = \{z | (\exists i_1, \cdots, i_m)(z = x_{i_1} \cdots x_{i_m} = y_{i_1} \cdots y_{i_m})\}$.

V. Prove the undecidability of the following problems.

1. Given $[M_1]$ and $[M_2]$ does there exist an infinite set, S, of inputs such that TM M_1 halts on each input in S and TM M_2 doesn't halt on any input in S?

2. Given a dlba M over the alphabet $\{a, b\}$, does there exist a value k such that $L(M) = \{x | \ x \in \{a, b\}^* \text{ and } k \leq |x| \leq 2k\}$? (Hint: Reduce the Post Correspondence Problem to this problem.)

VI. Design a P algorithm for the following problem. Estimate its speed.

Given an nfa M without ϵ-transitions, is $L(M)$ a finite set?

VII. Show that the following problem is NP-complete.

Given a CNF boolean expression, E, of length n and having at most $\sqrt{n}$ variables, is E satisfiable? (Hint: Make use of the fact that CNF SAT problem is NP-complete.)