600.271 Automata & Computation Theory
Final Examination
May 12, 2011
In-class, Closed Book
Time: 2 hrs 30 mins.

All the subproblems carry equal weight.

I. Design a deterministic pda for the language:
$\{a^n b^{n+2} c \mid n \geq 1\} \cup \{a^n b^{n+1} d \mid n \geq 1\} \cup \{a^n b^n \mid n \geq 1,\ n \text{ is an odd integer}\}$.

II. Design a CFG (i.e. type 2 grammar) for the language:

$\{x|\ x \in \{a,b\}^*,\ |x| \text{ is an even integer},\ x \neq x^R, \text{ and } \#_a x \text{ is an even integer}\}.$

III. Prove that the following problems are decidable.

1. Given a Post Correspondence Problem (x_1, y_1), $(x_2, y_2), \cdots, (x_n, y_n)$, $x_i, y_i \in \{0,1\}^+$, a dfa M, and a positive integer m, does there exist a string $z \in \{0,1\}^+$ of length no more than m such that $z \in L(M)$, and there exist $i_1, i_2, \cdots, i_k$ satisfying $x_{i_1} x_{i_2} \cdots x_{i_k} = y_{i_1} y_{i_2} \cdots y_{i_k} = z$?

2. Given a dlba M and positive integers m_1 and m_2, $m_1 < m_2$, do there exist strings x and y such that $m_1 \le |x|, |y| \le m_2$, $x \in L(M)$, and $y \notin L(M)$?

IV. Prove that the following set S is recursively enumerable by providing an appropriate enumeration algorithm:

$S = \{([M_1], [M_2]) |\ M_1$ and M_2 are TMs, and M_1 halts on blank tape$\}$.

V. Prove the undecidability of the following problems.

1. Given $[M]$ and a positive integer n, does TM M accept a string of length n and reject a string of length n? (Hint: Reduce the BTHP to this problem.)

2. Given deterministic pdas M_1 and M_2, and a dfa M_3, is $L(M_1) \cap L(M_2) \cap L(M_3) \neq \phi$? (Hint: Reduce the PCP to this problem.)

VI. Design a P algorithm for one of the following problems. Estimate its speed.

1. Given an undirected graph G of degree ≤ 2, compute the minimum size of its vertex cover. (Hint: Understand the structure of graphs when the degree is ≤ 2).

2. Given a directed graph G, vertices u and v, and a positive integer k, does G contain a path (need not be simple) from u to v of length at least k and at most $2k$?

3. Given a SAT expression E of length n in which at most $2 \log_2 n$ variables can appear as both uncomplemented and complemented, is E satisfiable?

VII. Show that the following problem is NP-complete.

Given a connected undirected graph G (with n vertices) and a positive integer $k \geq \frac{n}{2}$, does G contain a vertex cover of size k? (Hint: The standard vertex cover problem is NP-complete.)