## 600.271 Automata & Computation Theory Final Examination

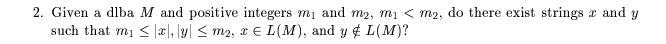
May 12, 2011 In-class, Closed Book Time: 2 hrs 30 mins.

All the subproblems carry equal weight.

I. Design a deterministic pda for the language:  $\{a^nb^{n+2}c|\ n\geq 1\}\cup\{a^nb^{n+1}d|\ n\geq 1\}\cup\{a^nb^n|\ n\geq 1,\ n\text{ is an odd integer}\}.$ 

II. Design a CFG (i.e. type 2 grammar) for the language:  $\{x|\ x\in\{a,b\}^*,\ |x|\ \text{is an even integer},\ x\neq x^R,\ \text{and}\ \#_ax\ \text{is an even integer}\}.$ 

- III. Prove that the following problems are decidable.
  - 1. Given a Post Correspondence Problem  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n), x_i, y_i \in \{0, 1\}^+$ , a dfa M, and a positive integer m, does there exist a string  $z \in \{0, 1\}^+$  of length no more than m such that  $z \in L(M)$ , and there exist  $i_1, i_2, \dots, i_k$  satisfying  $x_{i_1} x_{i_2} \dots x_{i_k} = y_{i_1} y_{i_2} \dots y_{i_k} = z$ ?



IV. Prove that the following set S is recursively enumerable by providing an appropriate enumeration algorithm:

 $S = \{([M_1], [M_2]) | M_1 \text{ and } M_2 \text{ are TMs, and } M_1 \text{ halts on blank tape}\}.$ 

- V. Prove the undecidability of the following problems.
  - 1. Given [M] and a positive integer n, does TM M accept a string of length n and reject a string of length n? (Hint: Reduce the BTHP to this problem.)

2. Given deterministic pdas  $M_1$  and  $M_2$ , and a dfa  $M_3$ , is  $L(M_1) \cap L(M_2) \cap L(M_3) \neq \phi$ ? (Hint: Reduce the PCP to this problem.)

VI. Design a P algorithm for one of the following problems. Estimate its speed.

- 1. Given an undirected graph G of degree  $\leq 2$ , compute the minimum size of its vertex cover. (Hint: Understand the structure of graphs when the degree is  $\leq 2$ ).
- 2. Given a directed graph G, vertices u and v, and a positive integer k, does G contain a path (need not be simple) from u to v of length at least k and at most 2k?
- 3. Given a SAT expression E of length n in which at most  $2\log_2 n$  variables can appear as both uncomplemented and complemented, is E satisfiable?

VII. Show that the following problem is NP-complete.

Given a connected undirected graph G (with n vertices) and a positive integer  $k \geq \frac{n}{2}$ , does G contain a vertex cover of size k? (Hint: The standard vertex cover problem is NP-complete.)