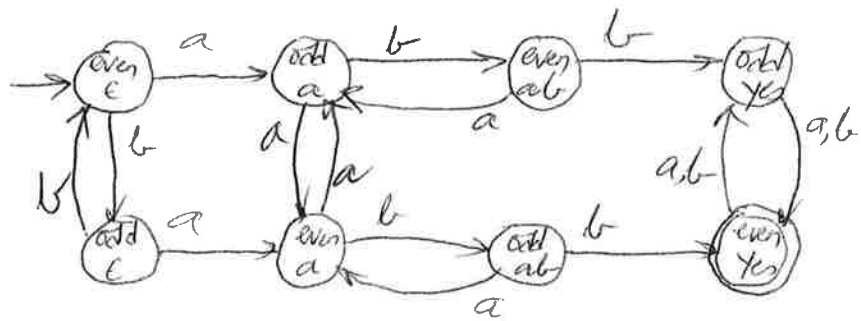
600.271 Automata & Computation Theory
Mid-Semester Examination
March 8, 2012
In-class, Closed Book, Time: 1 hr, 10 mins

All the subproblems carry equal weight. There are 5 subproblems in this examination.

I. Design the specified automaton for every one of the following languages.

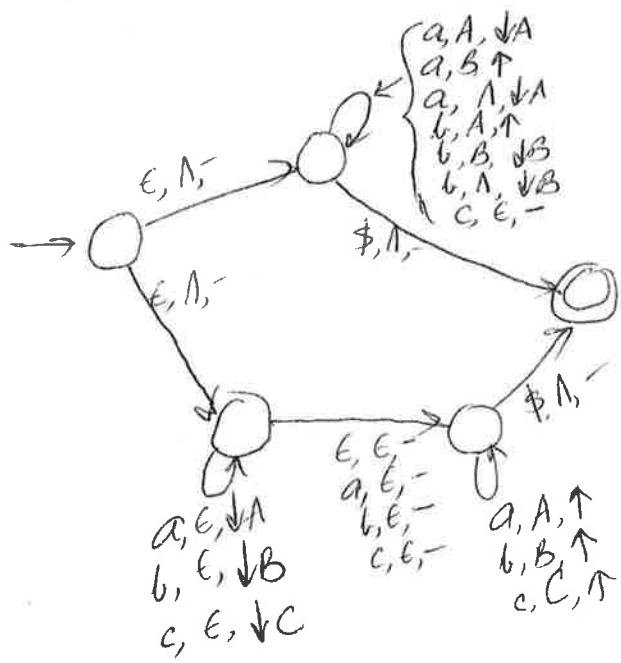
1. A dfa for the language $L_1 = \{x|\ x \in \{a,b\}^*,\ abb$ is a substring of x, and $|x|$ is an even integer$\}$.

Search for abb; also keep track of the parity of $|x|$.



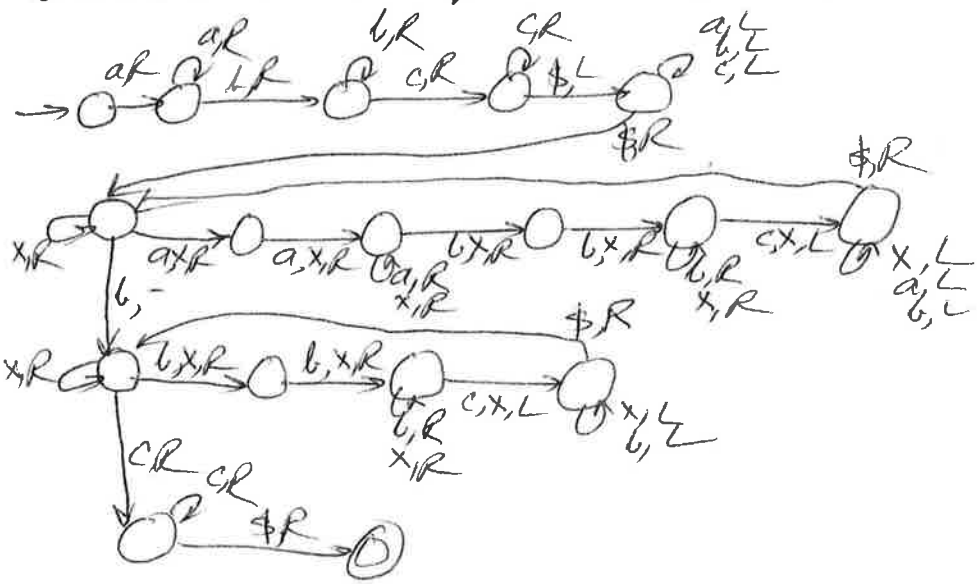
2. An npda for the language $L_2 = \{x|\ x \in \{a,b,c\}^*$, and $(\#_a x = \#_b x$, or $x = x^R\}$.

Guess the condition satisfied and verify. For $x = x^R$, guess the midpoint.



3. A dlba for the language $L_3 = \{a^i b^j c^k \mid i, j, k \geq 1,\ i \text{ and } j \text{ are even, and } i < j < 2k\}$.

check $x \in a^+b^+c^+$. cross off 2 a's, 2 b's, 1 c repeatedly until a's are exhausted. Then cross off 2 b's, 1 c repeatedly until b's are exhausted.



4. A CFG for the language $L_4 = \{a^i b^j a^k b^\ell \mid i, j, k, \ell \geq 1,$ and $(i = j$ and $k \neq \ell)$ or $(i = \ell$ and $j \neq k\}$.

From X generate $\{a^i b^i \mid i \geq 1\}$ and from Y generate $\{a^i b^j \mid i, j \geq 1, i \neq j\}$.

$X \to aXb \mid ab$

$Y \to U \mid V$

$U \to aU \mid aX$

$V \to Vb \mid Xb$

$S \to XY \mid W$ } $i = j$ and $k \neq l$ from XY ; $i = l, j \neq k$ from W

$W \to aWb \mid aYb$

II. Prove that the following language is not an fa language by applying the pumping lemma.
$L_5 = \{a^i b^j \mid i, j \geq 1, i \geq j \text{ or } 2i \leq j\}$.

For any k, let $z = a^{3k} b^{3k}$. Note that $z \in L_5$ since $3k \geq 3k$. Also $|z| \geq k$.

For any way of writing $a^{3k} b^{3k} = uvw$ s.t. $|uv| \leq k$ & $|v| > 0$:

since $|uv| \leq k$, $uv \in a^+$ & hence $v \in a^+$. Let $v = a^l$, $1 \leq l \leq k$.

Then $uv^0w = a^{3k-l} b^{3k} \notin L_5$.

Hence, by the contrapositive form of the PL, L_5 is not an fa language.