600.271 Automata & Computation Theory
Second Mid-Semester Examination
April 12, 2011
In-class, Closed Book
Time: 1 hr, 10 minutes

I. (10 pts.) Design a context-free grammar for the language
$\{xx^R yy^R |\ x, y \in \{a, b\}^+, abb \text{ is a substring of } x, \text{ and } |y| \text{ is odd}\}$

II. (10 pts.) Establish the decidability of one of the following problems by designing an appropriate algorithm.

- Given a CFG G, is there an $x \in L(G)$ such that $|x|$ is even?
- Given $[M_1]$ and $[M_2]$, M_1 and M_2 being dfa language recognizers, is $L(M_1) \cap L(M_2)$ an infinite set?

III. (10 pts.) Design a Turing machine for computing the following function.

$$f(x, y, z) = \begin{cases} x + (y - z) & \text{if } y \geq z \\ x & \text{otherwise} \end{cases}$$

IV. (10 pts.) Solve one of the following problems.

- Prove the undecidablity of the following problem. Given CFGs G_1 and G_2, is there an $x \in L(G_1) \cap (L(G_2))^R$, and $|x|$ is even? (Hint: Reduce the Post Correspondence Problem to this problem.

- Recall that the Uniform Halting Problem (UHP) asks whether a given Turing machine halts on every input. Reduce the UHP to the problem of testing whether a given TM computes the identity function; i.e. the function f s.t. $f(x) = x$ for every x.