

CS 461 - Computer Vision
 Professor Greg Hager
 Fall 2006 Homework 1 - Answer Key

a.) Given
$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1.4022 \\ 1 & -0.3457 & -0.7144 \\ 1 & 1.7710 & 0 \end{bmatrix} \begin{bmatrix} Y \\ Cb \\ Cr \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1.4022 \\ 1 & -0.3457 & -0.7144 \\ 1 & 1.7710 & 0 \end{bmatrix}^{-1} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} Y \\ Cb \\ Cr \end{bmatrix}$$

$$\begin{bmatrix} 0.29887 & 0.58662 & 0.11451 \\ -0.16876 & -0.33124 & 0.50000 \\ 0.50002 & -0.41836 & -8.1663 \times 10^{-2} \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} Y \\ Cb \\ Cr \end{bmatrix}$$

b.) Given $Cb = U - 0.5$ and $Cr = V - 0.5$

Substituting

$$\begin{bmatrix} 0.29887 & 0.58662 & 0.11451 \\ -0.16876 & -0.33124 & 0.50000 \\ 0.50002 & -0.41836 & -8.1663 \times 10^{-2} \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} Y \\ U - 0.5 \\ V - 0.5 \end{bmatrix}$$

$$\begin{bmatrix} 0.29887 & 0.58662 & 0.11451 \\ -0.16876 & -0.33124 & 0.50000 \\ 0.50002 & -0.41836 & -8.1663 \times 10^{-2} \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} Y \\ U \\ V \end{bmatrix} + \begin{bmatrix} 0 \\ -0.5 \\ -0.5 \end{bmatrix}$$

$$\begin{bmatrix} 0.29887 & 0.58662 & 0.11451 \\ -0.16876 & -0.33124 & 0.50000 \\ 0.50002 & -0.41836 & -8.1663 \times 10^{-2} \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} + \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} Y \\ U \\ V \end{bmatrix}$$

$$\begin{bmatrix} 0.29887 & 0.58662 & 0.11451 & 0 \\ -0.16876 & -0.33124 & 0.50000 & 0.5 \\ 0.50002 & -0.41836 & -8.1663 \times 10^{-2} & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \\ 1 \end{bmatrix} = \begin{bmatrix} Y \\ U \\ V \\ 1 \end{bmatrix}$$

c.) Given $R = G = B = x \quad x \in [0, 1]$

Substituting

$$\begin{bmatrix} 0.29887 & 0.58662 & 0.11451 & 0 \\ -0.16876 & -0.33124 & 0.50000 & 0.5 \\ 0.50002 & -0.41836 & -8.1663 \times 10^{-2} & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ x \\ x \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ 0.5 \\ 0.5 \\ 1 \end{bmatrix}$$

c.)

$Y \in [0, 1]$ for RGB values $[0, 0, 0]$ and $[1, 1, 1]$

$U \in [0, 1]$ for RGB values $[1, 1, 0]$ and $[0, 0, 1]$

$V \in [0, 1]$ for RGB values $[0, 1, 1]$ and $[1, 0, 0]$

$$\begin{bmatrix} 0.29887 & 0.58662 & 0.11451 & 0 \\ -0.16876 & -0.33124 & 0.50000 & 0.5 \\ 0.50002 & -0.41836 & -8.1663 \times 10^{-2} & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0.11451 & 0.58662 & 0.29887 & 0.70113 & 0.41338 & 0.88549 & 1.0 \\ 0.5 & 1.0 & 0.16876 & 0.33124 & 0.66876 & 0.83124 & 0 & 0.5 \\ 0.5 & 0.41834 & 0.08164 & 1.0 & -2.3 \times 10^{-5} & 0.91836 & 0.58166 & 0.50000 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

d.) $R \in [-0.7011, 1.7011]$ for YUV values $[0, x, 0]$ and $[1, x, 1]$

$G \in [-0.5300, 1.53]$ for YUV values $[0, 1, 1]$ and $[1, 0, 0]$

$B \in [-0.8855, 1.8855]$ for YUV values $[0, 0, x]$ and $[1, 1, x]$

$$\begin{bmatrix} 0.29887 & 0.58662 & 0.11451 & 0 \\ -0.16876 & -0.33124 & 0.50000 & 0.5 \\ 0.50002 & -0.41836 & -8.1663 \times 10^{-2} & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -0.70110 & 0.7011 & -0.7011 & 0.29891 & 0.70110 & 1.7011 & 0.2989 & 1.7011 \\ 0.53004 & -0.18435 & 0.18435 & 1.53 & -0.53004 & 0.81565 & 1.1843 & 0.46995 \\ -0.88549 & -0.88549 & 0.88549 & 0.11451 & 0.88549 & 0.11451 & 1.8855 & 1.8855 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

e.) It is not a good idea to convert from YUV to RGB before processing because of the truncation that occurs when you restrict the values to the unit cube.