CS 461 - Computer Vision
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Fall 2006 Homework 4 - Answer Key

1.)We have that the intrinsic parameter of both cameras are the same.

Where $f = 12.5mm$

$s_x = s_y = 0.01mm$

$o_x = 400$ pixels

$o_y = 300$ pixels

$$M_r = M_l = \begin{bmatrix} \frac{-f}{s_x} & 0 & o_x \\ 0 & \frac{-f}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-12.5}{0.01} & 0 & 400 \\ 0 & \frac{-12.5}{0.01} & 300 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1250.0 & 0 & 400 \\ 0 & -1250.0 & 300 \\ 0 & 0 & 1 \end{bmatrix}$$

First find the essential matrix, E

$E = RS$ where $R = I_{3\times3}$ $\quad S = sk(T) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -100 \\ 0 & 100 & 0 \end{bmatrix} \quad T = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$

$$E = IS = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -100 \\ 0 & 100 & 0 \end{bmatrix}$$

$$F = M_r^{-\intercal} E M_l^{-1}$$

$$\begin{bmatrix} -1250.0 & 0 & 400 \\ 0 & -1250.0 & 300 \\ 0 & 0 & 1 \end{bmatrix}^{-\intercal} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -100 \\ 0 & 100 & 0 \end{bmatrix} \begin{bmatrix} -1250.0 & 0 & 400 \\ 0 & -1250.0 & 300 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.08 \\ 0 & -0.08 & 0.0 \end{bmatrix}$$

2.)

a.) In our camera system the cameras are on the x-axis with a baseline distance d. They a verged at an angle θ, a rotation about the y-axis. Thus in the homogenous transform consists of R and t.

$$\text{Where } R = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \text{ and } t = \begin{bmatrix} d \\ 0 \\ 0 \end{bmatrix}$$

Forming the transform H we have

$$H = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & d \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b.)

$E = RS$ where R is same an above and $S = sk(t)$ with t from above.

Substituting

$$E = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -d \\ 0 & d & 0 \end{bmatrix} = \begin{bmatrix} 0 & -d\sin\theta & 0 \\ 0 & 0 & -d \\ 0 & d\cos\theta & 0 \end{bmatrix}$$

3.) A homography is a matrix forming the relationship between the points on 2 plane. Given points p_1 and p_2 from 2 planes we have

$$p_1 = Hp_2$$

We know that the relative pose of a point,p, on the plane,P, in the 2 cameras is given by

$$X_2 = RX_1 + T$$

where X_1 and X_2 are the coordinates of p relative to the 2 camera frames.

Let N be the unit normal vector of the plane P, which respect to the first camera, and let $d > 0$ be the distance from the plane to the optical center of the the first camera. Thus $N^\intercal X_1 = d$ describes the plane.

$$\Rightarrow \frac{1}{d} N^\intercal X_1 = 1 \qquad \forall X_1 \in P$$

Substituting

$$\begin{aligned} X_2 &= RX_1 + T\left(\frac{1}{d} N^\intercal X_1\right) \\ &= \left(R + T\frac{1}{d} N^\intercal\right) X_1 \end{aligned}$$

$$X_2 = HX_1$$

Where R is rotation, T is translation. Solving for H we have the homography

$$H = R + \frac{1}{d}TN^{\intercal}$$

4.)The first point where is easiest to see makes the disparity equal to zero is the point which is equidistant from the projective centers. In this case, because of the symmetry about the z-axis the disparity is zero. Similarly when points found the same angle to the optical centers of the camera the disparity is also zero from similar triangles. Plotting all such point forms a center of the horopters this circle is called the Vieth-Muller circle and is defined by the projective center and the baseline distance. Given O_c the point at the intersection of the optical centers and O_r and O_l the points at the projective centers of the right and left cameras respectively. The circle is formed as $2\alpha = \measuredangle O_l O_c O_r$ and $2d$ is the baseline distance.

Thus

$$r \sin\alpha = d \Rightarrow r = \frac{d}{\sin\alpha}$$

$$O_c = O_l r \sin\alpha = (O_r \frac{d}{\tan\alpha})$$

$$x^2 + (z - \frac{d}{\tan\alpha})^2 = \frac{d^2}{\sin^2\alpha}$$